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RESEARCH ARTICLE

Variational Iteration Method for Burger's Equation in Different Positions

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ABSTRACT

The method of variational iteration was found to pave an exact way to solve the Burger's equation at various locations. The Variational Iteration Method (VIM) was proved to be efficacious, potent, straightforward and not complicated, when matched with various techniques. This made this method to exhibit its implementation in all aspects of problems which fall under physics as well as problems related to mathematics, linear and nonlinear ones.

Keywords: VIM, Mathematical problems, Linear equations, Nonlinear equations, Burger's equation.

1. INTRODUCTION

While research imply that nonlinear equations are among the supreme phenomenon around the globe, the nonlinearity has its vital reaction on applied mathematics, physics, and matters regarding engineering fields. Here the dissimilarity of every parameter relies on various features. The significance of procuring the precise or estimated results of Nonlinear Partial Differential Equations (NLPDEs) in physical and mathematical problems is even now problematic that deserves novel techniques to uncover novel, precise or estimated results. The nonlinear equations entirely never usually possess an exact analytical solution, but there are certain analytical methods which does so. Perturbation techniques and the method of d-expansion are certain standard analytical methods. Recently, several authors are focusing on the study results that are obtained from the NLPDE by the aid of VIM.

The method of VIM, that was suggested initially by[1-4], was evinced by several authors to exist as a potent tool for mathematics when it comes for problems like linear and nonlinear ones [5-15]. In contrast to the old-style numerical methods, the features of discretization, linearization, transformation or perturbation is no longer in need for VIM. This method has been utilized to gain solutions to problems that are of nonlinear type. Implementation of VIM, has expanded because of the features of flexibility, handiness and accurateness, they possess. The implementations of the VIM methods are illustrated in [16-20]. This work, suggests that in various locations, Burger's equation can be solved precisely by the method of VIM. The analysis is shown in equations (1) to (21). The equations that govern are,

$$\frac{\partial\phi}{\partial t}+\alpha\left(\phi\frac{\partial\phi}{\partial x}+\phi\frac{\partial\phi}{\partial y}\right)-\beta\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)=0, \qquad (2+1)-dimensional \tag{1}$$

$$\frac{\partial\phi}{\partial t}+\alpha\left(\phi\frac{\partial\phi}{\partial x}+\phi\frac{\partial\phi}{\partial y}+\phi\frac{\partial\phi}{\partial z}\right)-\beta\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)=0, \qquad (3+1)-dimensional \tag{2}$$

$$\frac{\partial\phi}{\partial t}+\alpha\left(\phi\frac{\partial\phi}{\partial x_1}+\phi\frac{\partial\phi}{\partial x_2}+\cdots+\phi\frac{\partial\phi}{\partial x_n}\right)-\beta\left(\frac{\partial^2\phi}{\partial x_1^2}+\frac{\partial^2\phi}{\partial x_2^2}+\cdots+\frac{\partial^2\phi}{\partial x_n^2}\right)=0, \qquad (n+1)-dimensional \tag{3}$$

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2. BASIC IDEA OF VIM

The notion of VIM is founded on using a common Lagrange multiplier to create a correction solution. This multiplier is selected as though its solution for correction is enhanced with reverence to either the primary approximation or the experimental function.

To elucidate the theme of He's VIM, the subsequent differential equations are deliberated:

$$\pounds\phi + N\phi = g(t) \tag{4}$$

Here £ is a linear operator, N is a nonlinear operator and $g(t)$ an inhomogeneous term. In accordance with VIM, the correction functional can be written as,

$$\phi_{n+1}(t) = \phi_n(t) + \int_0^t \lambda(\tau)\left[\pounds\phi_n(\tau) + N\widetilde{\phi}_n(\tau) - g(\tau)\right] d\tau, \qquad n \geq 0, \tag{5}$$

Here λ is a common Lagrange multiplier. The variational theory aids to find the Lagrange multiplier. The subscript k signifies kth-order approximation and $\widetilde{\phi}_k$ is deliberated as a controlled variation i.e. $\delta\widetilde{\phi}_n = 0$.

The theoretical explanation given above is demonstrated with various examples like Burger's equation in various locations, and the solutions that were achieved were matched with [21].

3. APPLICATIONS

Problem 3.1: (2+1)-dimensional Burger's equation

The first form of the Burger's equation is the (2+1)-dimensional Burger's equation defined as (1) with the primary condition,

$$\phi(x,y,0) = x + y \tag{6}$$

The correction functional for equation (1) in the t-direction possesses the form,

$$\phi_{k+1}(x,y,t) = \phi_k(x,y,t) + \int_0^t \lambda(\tau)\left[\frac{\partial\phi_k}{\partial\tau} + \alpha\left(\phi\frac{\partial\widetilde{\phi}_k}{\partial x} + \phi\frac{\partial\widetilde{\phi}_k}{\partial y}\right) - \beta\left(\frac{\partial^2\widetilde{\phi}_k}{\partial x^2} + \frac{\partial^2\widetilde{\phi}_k}{\partial y^2}\right)\right] d\tau \tag{7}$$

The stationary conditions for equation (7) are,

$$\lambda'(\tau) = 0, \qquad 1 + \lambda(\tau)|_{\tau=t} = 0 \tag{8}$$

This in turn gives,

$$\lambda = -1 \tag{9}$$

When this Lagrange multiplier value is substituted into the functional (7), it produces the following iteration formula,

$$\phi_{k+1}(x,y,t) = \phi_k(x,y,t) - \int_0^t \left[\frac{\partial\phi_k}{\partial\tau} + \alpha\left(\phi\frac{\partial\phi_k}{\partial x} + \phi\frac{\partial\phi_k}{\partial y}\right) - \beta\left(\frac{\partial^2\phi_k}{\partial x^2} + \frac{\partial^2\phi_k}{\partial y^2}\right)\right] d\tau \tag{10}$$

Keeping the condition (6) in mind, we can choose $\phi_0(x,y,t) = \phi_0(x,y,0) = x + y$. By utilizing this selected value in equation (10) the following first approximation can be achieved.

$$\phi_1(x,y,t) = \phi_0(x,y,t) - \int_0^t [0 + \alpha\{(x+y)(1) + (x+y)(1)\} - 0]\, d\tau$$

$$= \phi_0(x,y,t) - \int_0^t [2\alpha(x+y)]\, d\tau = (x+y)(1-2\alpha t)$$

Similarly, the following successive approximations can be achieved.

$$\phi_0(x,y,t) = x + y$$

$$\phi_1(x,y,t) = (x+y)(1-2\alpha t)$$

$$\phi_2(x,y,t) = (x+y)\left[1-2\alpha t+4\alpha^2t^2+O(t^3)\right]$$
$$\phi_3(x,y,t) = (x+y)\left[1-2\alpha t+4\alpha^2t^2-8\alpha^3t^3+O(t^4)\right]$$
$$\vdots$$
$$\phi_n(x,y,t) = (x+y)(1-2\alpha t+4\alpha^2t^2-8\alpha^3t^3+16\alpha^4t^4-32\alpha^5t^5+\cdots)$$

In the closed form this produces the solution as,

$$\phi(x,y,t) = \frac{x+y}{1+2\alpha t} \tag{11}$$

This solution is the precise solution that fulfills the (2+1)-dimensional Burger's equation (equation (1)) and condition (6) and same as that of [21].

Problem 3.2: (3+1) - dimensional Burger's equation

The next, which is the second form of the Burger's equation is the (3+1)-dimensional Burger's equation defined as (2) with the primary condition,

$$\phi(x,y,z,0) = x+y+z \tag{12}$$

The correction functional for equation (2) in the t-direction possesses the form,

$$\phi_{k+1}(x,y,z,t) = \phi_k(x,y,z,t)+$$
$$\int_0^t \lambda(\tau)\left[\frac{\partial\phi_k}{\partial\tau}+\alpha\left(\phi\frac{\partial\widetilde{\phi}_k}{\partial x}+\phi\frac{\partial\widetilde{\phi}_k}{\partial y}+\phi\frac{\partial\widetilde{\phi}_k}{\partial z}\right)-\beta\left(\frac{\partial^2\widetilde{\phi}_k}{\partial x^2}+\frac{\partial^2\widetilde{\phi}_k}{\partial y^2}+\frac{\partial^2\widetilde{\phi}_k}{\partial z^2}\right)\right]d\tau \tag{13}$$

The stationary conditions give,

$$\lambda = -1 \tag{14}$$

When this Lagrange multiplier value is substituted into the functional (13), it produces the following iteration formula,

$$\phi_{k+1}(x,y,z,t) = \phi_k(x,y,z,t)-$$
$$\int_0^t \left[\frac{\partial\phi_k}{\partial\tau}+\alpha\left(\phi\frac{\partial\phi_k}{\partial\tau}+\phi\frac{\partial\phi_k}{\partial y}+\phi\frac{\partial\phi_k}{\partial z}\right)-\beta\left(\frac{\partial^2\phi_k}{\partial x^2}+\frac{\partial^2\phi_k}{\partial y^2}+\frac{\partial^2\phi_k}{\partial z^2}\right)\right]d\tau \tag{15}$$

Considering the given condition (12), we can choose $\phi_0(x,y,z,t) = \phi_0(x,y,z,0) = x+y+z$. By utilizing this selected value in (15) the following first approximation can be attained.

$$\phi_1(x,y,z,t) = \phi_0(x,y,z,t) - \int_0^t \left[0+\alpha\left\{(x+y+z)(1)+(x+y+z)(1)+(x+y+z)(1)\right\}-0\right]d\tau$$

$$= \phi_0(x,y,z,t) - \int_0^t \left[3\alpha(x+y+z)\right]\partial\tau = (x+y+z)(1-3\alpha t)$$

Similarly, the following successive approximations can be achieved.

$$\phi_0(x,y,z,t) = x+y+z$$
$$\phi_1(x,y,z,t) = (x+y+z)(1-3\alpha t)$$
$$\phi_2(x,y,z,t) = (x+y+z)\left[1-3\alpha t+9\alpha^2t^2+O(t^3)\right]$$
$$\phi_3(x,y,z,t) = (x+y+z)\left[1-3\alpha t+9\alpha^2t^2-27\alpha^3t^3+O(t^4)\right]$$
$$\vdots$$

$$\phi_n(x,y,z,t) = (x+y+z)(1-3\alpha t+9\alpha^2 t^2-27\alpha^3 t^3+81\alpha^4 t^4-243\alpha^5 t^5+\cdots)$$

In closed form this produces the solution,

$$\phi(x,y,z,t) = \frac{x+y+z}{1+3\alpha t} \tag{16}$$

This solution is the precise solution that fulfills the (3+1)-dimensional Burger's equation (equation (2)) and condition (12) and same as that of [21].

Problem 3.3: (n+1)-dimensional Burger's equation

The final form of the Burger's equation is the (n+1)-dimensional Burger's equation defined as (3) with the initial condition,

$$\phi(x_1,x_2,...,x_n,0) = x_1+x_2+...+x_n \tag{17}$$

The correction functional for equation (3) in the t-direction possesses the form,

$$\phi_{k+1}(x_1,x_2,...,x_n,0) = \phi_k(x_1,x_2,\cdots,x_n,0)+$$
$$\int_0^t \lambda(\tau)\left[\frac{\partial\phi_k}{\partial\tau}+\alpha\left(\phi\frac{\partial\widetilde{\phi}_k}{\partial x_1}+\phi\frac{\partial\widetilde{\phi}_k}{\partial x_2}+\cdots+\phi\frac{\partial\widetilde{\phi}_k}{\partial x_n}\right)-\beta\left(\frac{\partial^2\widetilde{\phi}_k}{\partial x_1^2}+\frac{\partial^2\widetilde{\phi}_k}{\partial x_2^2}+\cdots+\frac{\partial^2\widetilde{\phi}_k}{\partial x_n^2}\right)\right]d\tau \tag{18}$$

The stationary conditions give,

$$\lambda = -1 \tag{19}$$

When this Lagrange multiplier value is substituted into the functional (21) it produces the following iteration formula,

$$\phi_{k+1}(x_1,x_2,...,x_n,0) = \phi_k(x_1,x_2,...,x_n,0)-$$
$$\int_0^t\left[\frac{\partial\phi_k}{\partial\tau}+\alpha\left(\phi\frac{\partial\phi_k}{\partial x_1}+\phi\frac{\partial\phi_k}{\partial x_2}+\cdots+\phi\frac{\partial\phi_k}{\partial x_n}\right)-\beta\left(\frac{\partial^2\phi_k}{\partial x_1^2}+\frac{\partial^2\phi_k}{\partial x_2^2}+\frac{\partial^2\phi_k}{\partial x_n^2}\right)\right]d\tau \tag{20}$$

Considering the given condition (17), we can choose $\phi_0(x_1,...,x_n,t) = \phi_0(x_1,...,x_n,0) = x_1+x_2+\cdots+x_n$. By utilizing this selected value in (20), the following first approximation can be attained.

$$\phi_1(x_1,x_2,...,x_n,0) = \phi_0(x_1,x_2,...,x_n,0)-$$
$$\int_0^t [0+\alpha\{(x_1+x_2+\cdots+x_n)(1)+(x_1+x_2+\cdots+x_n)(1)+\cdots+(x_1+x_2+\cdots+x_n)(1)\}-0]\,d\tau$$
$$\phi_0(x_1,x_2,\cdots,x_n,0)-\int_0^t[n\alpha(x_1+x_2+\cdots+x_n)]\,\partial\tau = (x_1+x_2+\cdots+x_n)(1-n\alpha t)$$

Similarly, the following successive approximations can be obtained.

$$\phi_0(x_1,x_2,...,x_n,t) = x_1+x_2+...+x_n$$
$$\phi_1(x_1,x_2,...,x_n,t)+(x_1+x_2+\cdots+x_n)(1-n\alpha t)$$
$$\phi_2(x_1,x_2,...,x_n,t) = (x_1+x_2+\cdots+x_n)\left[1-n\alpha t+n^2\alpha^2t^2+O(t^3)\right]$$
$$\phi_3(x_1,x_2,...,x_n,t) = (x_1+x_2+\cdots+x_n)\left[1-n\alpha t+n^2\alpha^2t^2-n^3\alpha^3t^3+O(t^4)\right]$$
$$\vdots$$
$$\phi_n(x_1,x_2,...,x_n,t) = (x_1+x_2+\cdots+x_n)(1-n\alpha t+n^2\alpha^2t^2-n^3\alpha^3t^3+\cdots)$$

In closed form this produces the solution.

$$\phi(x_1,x_2,...,x_n,t) = \frac{x_1+x_2+\cdots+x_n}{1+n\alpha t} \tag{21}$$

This solution is the precise solution that fulfills the (n+1)-dimensional Burger's equation (equation (3)) and condition (17) and is same as that of [21].

4. CONCLUSION

In this work, the method of VIM was efficaciously used to discover Burger's equation's solution. The result thus attained by making use of the VIM method, is actually an infinite series for suitable primary conditions. This can further be conveyed in a closed form to produce the anticipated precise solution. The clarity of VIM and the results got, reveal that VIM is a potent tool in mathematics to solve the Burger's equation and also an encouraging technique to solve various equations which are linear and nonlinear.

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